2022 Speed Round Solutions

Key (solutions start on next page)

1. 0
2. 58
3. 6
4. 50
5. 3
6. 30
7. 1000
8. 37
9. 54
10. 4
11. 18
12. 2022
13. 92
14. 10
15. 60
16. 15
17. 91
18. 81
19. 1
20. 38
21. 18
22. 5
23. 4
24. 8
25. 48
26. 5
27. 1728
28. 4
29. 28
30. 41

Solutions

1. What is the value of the expression $2 - \frac{2}{2-\frac{2}{2}}$?

   Answer: $\boxed{0}$.

   Solution: Evaluate the expression as $2 - \frac{2}{2-1} = 2 - \frac{2}{1} = 2 - 2 = 0$. $\square$

2. Willow counts downwards by 7's, starting from $100, 93, 86, \ldots$. What is the 7th number she says?

   Answer: $\boxed{58}$.

   Solution: The 2nd number she says is less than 100 by 7×1; the 3rd number she says is less than 100 by 7×2; and so on. Hence the 7th number she says is less than 100 by $7 \times 6 = 42$, i.e. the number is $100 - 42 = 58$. $\square$

3. Gabriela has a basket containing apples and pears, and she has twice as many pears as apples. If the basket contains 9 fruits in total, how many pears are there?

   Answer: $\boxed{6}$.

   Solution: Since she has two pears for every apple, one-third of the fruits must be apples. So there are 3 apples, and 6 pears. $\square$

4. 30% of 50 is equal to what percent of 30?

   Answer: $\boxed{50}$.

   Solution: 30% of 50 is equal to $0.3 \cdot 50 = 15$. This is half of 30, so the answer is 50%. $\square$

5. Jeffrey spent \$20 on oranges, clementines, and grapefruits. Each of the fruits costs \$2 apiece. If Jeffrey bought 3 oranges and 4 clementines, how many grapefruits did he buy?

   Answer: $\boxed{3}$.

   Solution: The amount of money Jeffrey spent on oranges and clementines is $3 \cdot \$2 + 4 \cdot \$2 = \$14$. The remaining \$6 he spent on grapefruits, so he must have bought 3 grapefruits. $\square$

6. How many two digit numbers are divisible by 3?

   Answer: $\boxed{30}$.

   Solution: The numbers in question are $12, 15, 18, \ldots, 99$. We can write these numbers as $3 \cdot 4, 3 \cdot 5, 3 \cdot 6, \ldots, 3 \cdot 33$. So we want to count how many numbers are in the range $4, 5, 6, \ldots, 33$. There are 30 of these numbers (it's the first 33 numbers, but missing 1, 2, and 3). $\square$

7. Isaac and Eli are each trying to approximate the value of the sum $17622 + 43721$.

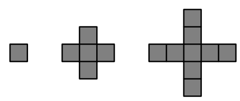
   - Isaac rounds both of the numbers to the nearest thousand, and then adds them together.
   - Eli adds the numbers together, and then rounds the result to the nearest thousand.

   What is the positive difference between Isaac's result and Eli's result?

   Answer: $\boxed{1000}$.

   Solution: Isaac first rounds to the numbers 18000 and 44000, and gets the sum as 62000. On the other hand, Eli first adds them to get 61343, and then rounds to 61000. The difference between the two results is 1000. □

8. Consider a sequence of shapes, the first three of which are as follows:



   How many squares are in the tenth shape?

   Answer: $\boxed{37}$.

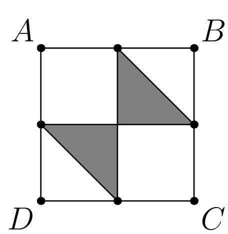
   Solution: Notice that at each step, we add four more squares. Hence in the n-th shape, we have $4n - 3$ squares. This means the 10th shape has $4 \cdot 10 - 3 = 37$ squares. □

9. At NCSSM, 3 cups of boba are worth 2 bags of ramen, and 1 bag of ramen is worth 9 ping pong balls. How many ping pong balls are 9 cups of boba worth?

   Answer: $\boxed{54}$.

   Solution: Since 3 cups of boba are worth 2 bags of ramen, we must have that 9 cups of boba are worth 6 bags of ramen. These 6 bags of ramen are worth $6 \cdot 9 = 54$ ping pong balls. □

10. Jonathan draws a square $ABCD$ of side length 4, and then marks the midpoints of the four sides as shown. What is the total area of the shaded region?



   Answer: $\boxed{4}$.

   Solution: Note that we can piece together the two shaded triangles to form a square, of side length 2. Hence the total shaded area is $2^2 = 4$. □

11. The numbers $3, 4, 5, 6$ each go in one of the four blanks below:

$$\_\_\_ \times \_\_\_ - \_\_\_ \times \_\_\_$$

What is the largest possible value of the above expression, over all ways to fill in the blanks?

Answer: $\boxed{18}$. We want to make the first half of the expression as big as possible, and the second half as small as possible. This is accomplished by setting $6 \times 5 - 4 \times 3 = 30 - 12 = 18$.

12. If 2022 workers can dig 2022 holes in 2022 days, how many days does it take 2023 workers to dig 2023 holes?

Answer: $\boxed{2022}$.

Solution: Each worker digs one hole in 2022 days. Hence it takes 2022 days for 2023 workers to dig 2023 holes. $\square$

13. Philip took four tests, and his average score was 90. If he scores 100 on his next test, what will his average score be across the five tests?

Answer: $\boxed{92}$.

Solution: The sum of all of Philips scores so far must be $90 \cdot 4 = 360$. If he scores a 100 on his next test, the sum of all his scores across the 5 tests will be 460. Then his average score across the 5 tests will be $460/5 = 92$. $\square$

14. Angie and Bonnie are 100 meters apart from each other, and running towards each other. Angie runs at a speed of 4 meters per second, and Bonnie runs at a speed of 6 meters per second. How many seconds will it take for them to run into each other?

Answer: $\boxed{10}$.

Solution: The rate at which the distance between them closes is $4\text{m/s} + 6\text{m/s} = 10\text{m/s}$. Hence it takes $100/10 = 10$ seconds before the distance between them is zero. $\square$

15. Isaac buys a ruler and a compass for a total price of \$2.20. If the compass cost \$1.00 more than the ruler, how many cents did the ruler cost?

Answer: $\boxed{60}$.

Solution: If the cost of the ruler is r, and the cost of the compass is $1 + r$, then we have $1 + 2r = 2.2 \implies r = 0.6$. $\square$

16. Kevin owns 12 buffalo, 3 of which are albino. Nathan owns 24 buffalo, some of which are also albino. Kevin and Nathan notice that when all their buffalo are combined, 50% of them are albino. How many of Nathan's buffalo are albino?

Answer: $\boxed{15}$.

Solution: In total, there are 36 buffalo, and 50% of them are albino, i.e. 18 buffalo. Since Kevin has 3 of them, this means Nathan must have the remaining 15. $\square$

17. Two old cat ladies live in a house together. The first cat lady has 10 adult cats, each of which has 10 kittens, each of which has 10 toys. The second cat lady has 9 adult cats, each of which has 9 kittens, each of which has 9 toys.

    One day, all of the kittens misbehaved, so the adult cats confiscated all of the toys and distributed them among themselves. On average, how many toys went to each adult cat?

    Answer: $\boxed{91}$.

    Solution: The total number of toys is $10 \times 10 \times 10 + 9 \times 9 \times 9 = 1729$. There are 19 total adult cats, so on average each of the adults has $1729/19 = 91$ toys. $\square$

18. Out of the 500 students at NCSSM, 258 are Juniors and 242 are Seniors. 342 students like math, while the other 158 of them don't. If 181 of the Juniors like math, how many of the Seniors don't like math?

    Answer: $\boxed{81}$.

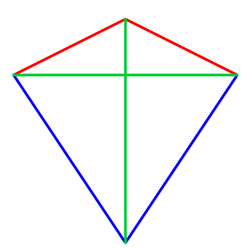
    Solution: If 181 of the Juniors like math, the other $258 - 181 = 77$ of them do not like math. Out of the 158 people who don't like math, the remaining $158 - 77 = 81$ of them must be Seniors. $\square$

19. A quadrilateral $ABCD$ has $AB = BC, CD = DA$, and $AC = BD$. How many of the following statements must be true?

    - The diagonals of the quadrilateral are perpendicular.
    - All four sides of the quadrilateral are equal.
    - The quadrilateral is a parallelogram.
    - The quadrilateral is a square.

    Answer: $\boxed{1}$.

    Solution: The shape must be a kite, so the first statement must be true. The other three statements don't have to be true, as shown below.



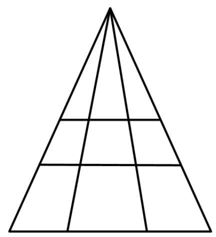
$\square$

20. In a basketball game, Eli attempted four times as many 2-pointer shots as 3-pointer shots. In addition, his accuracy on 3-pointer shots was half his accuracy on 2-pointer shots. If Eli scored 32 points from making 2-pointer shots, how many total points did he score in the game?

Answer: $\boxed{38}$.

Solution: Since Eli attempted four times as many 2-pointers as 3-pointers, and had twice as high of an accuracy, Eli must have made 8 times as many 2-pointers as 3-pointers. This means that since Eli made 16 shots that were 2-pointers, he must have made 2 shots that were 3-pointers. So his total score is $16 \cdot 2 + 2 \cdot 3 = 38$. ☐

21. How many triangles are there in the following image?



Answer: $\boxed{18}$.

Solution: One of the sides of the triangle must be a horizontal segment. Once we pick this segment, the rest of the triangle is determined. We see there are 18 possible horizontal segments, so there are 18 total triangles. ☐

22. I have enough money to buy 12 apples. This amount of money is also exactly enough to buy 60 grapes. A recipe that makes 2 fruit salads calls for 2 apples and 14 grapes. How many fruit salads can I make?

Answer: $\boxed{5}$.

Solution: Let the amount of money I have be m: then the cost of an apple must be $m/12$, and the cost of a grape must be $m/60$. This means the cost of 2 salads is $2\frac{m}{12} + 14\frac{m}{60} = \frac{24}{60}m = \frac{2}{5}m$. So a single salad costs $\frac{1}{5}m$. Hence since the amount of money I have is m, I can make 5 salads. ☐

23. Alice and Bob are playing a guessing game. Alice picks a point in a two-dimensional space. Next, Bob guesses what he thinks this point is. Alice then tells Bob the distance between his point and the correct point. Assuming Bob guesses with a good strategy, what is the maximum number of guesses it could take for Bob to guess the correct point?

Answer: $\boxed{4}$.

Solution: After Bob's first guess, Bob knows that Alice's point is somewhere on a certain circle. Next, Bob guesses a point on that circle: when Alice tells him the distance, it restricts the possibilities down to two points on the circle. It then takes Bob at most two guesses to guess the correct one out of these points. In total, Bob has made 4 guesses. ☐

24. Sam, Jacob, and Kyle have the integers $1, 2, 3, \ldots, 30$ written on a whiteboard. They then erase some of the numbers as follows:

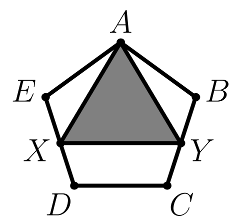
- Sam erases all the multiples of 2 (i.e. $2, 4, 6, \ldots, 30$).
- Of the remaining numbers, Jacob erases all the multiples of 3.
- Of the remaining numbers, Kyle erases all the multiples of 5.

How many integers remain on the whiteboard?

Answer: $\boxed{8}$.

Solution: When picking a random number from 1 through 30, there is a $\frac{1}{2}$ chance that it's not divisible by 2, a $\frac{2}{3}$ chance that it's not divisible by 3, and a $\frac{4}{5}$ chance that it's not divisible by 5. These probabilities are all independent.[1] Hence there's overall a $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{4}{5} = \frac{8}{30}$ chance that the number will not be erased. This implies that the answer is 8. $\square$

25. $ABCDE$ is a regular pentagon, as shown. Points X and Y are on sides BC and DE such that $\triangle AXY$ is equilateral, and line XY is parallel to line CD. What is the measure of $\angle AXE$ in degrees?



Answer: $\boxed{48}$.

Solution: First, recall that the angles in a pentagon are $108°$. Since lines XY and CD are parallel, we have $\angle YXD = 180° - 108° = 72°$. Now we have

$$\angle AXE = 180° - \angle AXY - \angle YXD = 180° - 60° - 72° = 48°.$$

$\square$

26. Five suspects, one of which committed a crime, are being investigated. They give the following statements:

- Suspect 1: "It was either suspect 2 or 5."
- Suspect 2: "It was neither me nor suspect 3."
- Suspect 3: "Whatever suspect 4 says next is going to be incorrect."
- Suspect 4: "Out of suspects 1 and 2, one of them told the truth and the other lied."
- Suspect 5: "No, both suspects 1 and 2 lied."

It is also known that exactly two of the suspects are lying. Which suspect committed the crime?

Answer: $\boxed{5}$.

Solution: Note that exactly one of suspects 3 and 4 must be lying, due to suspect 3's statement. Thus suspect 5's statement cannot be true, or else there would be too many liars. Hence suspects 1 and 2 are both telling the truth, i.e. suspect 5 was the one who committed the crime. $\square$

27. Let $n = 2022^{22}$. How many positive integers m are there such that m is a divisor of n, and n is a divisor of m^2?

Answer: $\boxed{1728}$.

Solution: Since the prime factorization of 2022^{22} is $2^{22} \cdot 3^{22} \cdot 337^{22}$, we can let $m = 2^a \cdot 3^b \cdot 337^c$ for some nonnegative integers $a, b, c, \leq 22$. In addition, we wish to have $2a, 2b, 2c \geq 22 \implies a, b, c \geq 11$. This means we have 12 options to pick for each of a, b, c, giving a total of $12^3 = 1728$ options. $\square$

[1] see https://en.wikipedia.org/wiki/Chinese_remainder_theorem

28. Isaac, Albert, and Eli are attempting to guess Hari's secret number, which is a random integer from 1 to 12. Isaac guesses first; if he doesn't get it, then Albert guesses; if neither of them gets it, then Eli guesses; if the number still has not been guessed, then it goes back to Isaac, and the cycle continues. Nobody ever guesses a number that has already been guessed. What is the probability that Isaac is the first to guess the secret number? Say that the probability is $\frac{a}{b}$ when written in lowest terms (i.e. when a and b are relatively prime). Submit the number $a+b$.

Answer: $\boxed{4}$.

Solution: Imagine if Hari never says anything about whether the guesses are correct or not, and the players simply each guess four times until all 12 numbers are gone. In terms of information the players get, this is equivalent to the original game. However, since the players are now just each picking a list of four numbers, each of them is equally likely to pick the winning number. So the answer is 1/3, or 4. □

29. How many of the integers from 1 to 1000 are perfect squares but not perfect cubes?

Answer: $\boxed{28}$.

Solution: The squares from 1 to 1000 are $1^2, 2^2, 3^2, \ldots, 31^2$. A number is both a perfect square and a perfect cube if and only if it is a perfect 6th power. The perfect 6th powers less than 1000 are $1^6, 2^6, 3^6$. Hence we must subtract these out, ending up with a total of $31 - 3 = 28$. □

30. Holden creates a sequence of numbers by starting with $a_1 = 2021$ and setting $a_n = 20 + a_{n-1}$ if n is even, and $a_n = 21a_{n-1}$ if n is odd. What are the last two digits of a_{2022}?

Answer: $\boxed{41}$.

Solution: We only need to look at the last two digits of any of the numbers. It can be checked that these last two digits cycle as $21 \to 41 \to 61 \to 81 \to 01 \to 21 \to \cdots$. This means that a_{n+5} has the same last two digits as a_n. Hence a_{2022} has the same last two digits as a_2, which has last two digits 41. □